



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

TRANSIENT RESPONSE ANALYSIS

UNDERDAMPED SECOND ORDER SYSTEM-II

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RESPONSE OF AN UNDERDAMPED SECOND ORDER SYSTEM

PEAK TIME (T_p)

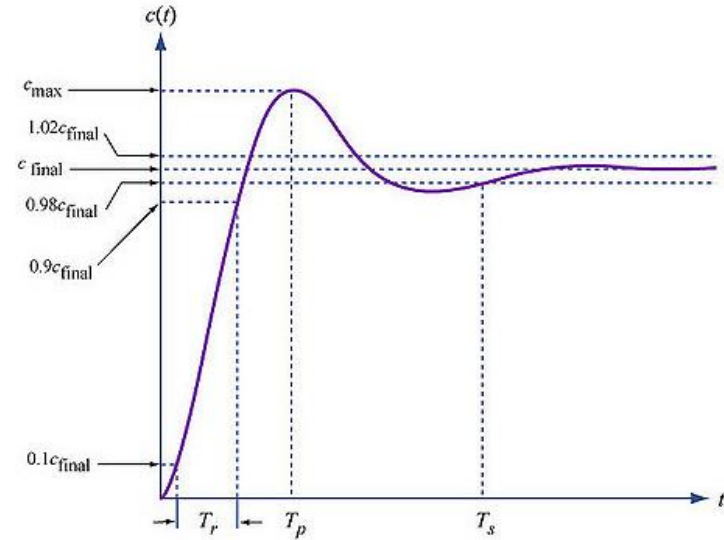
Time required for the response to reach the first, or maximum peak

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

PERCENT OVERSHOOT (%OS)

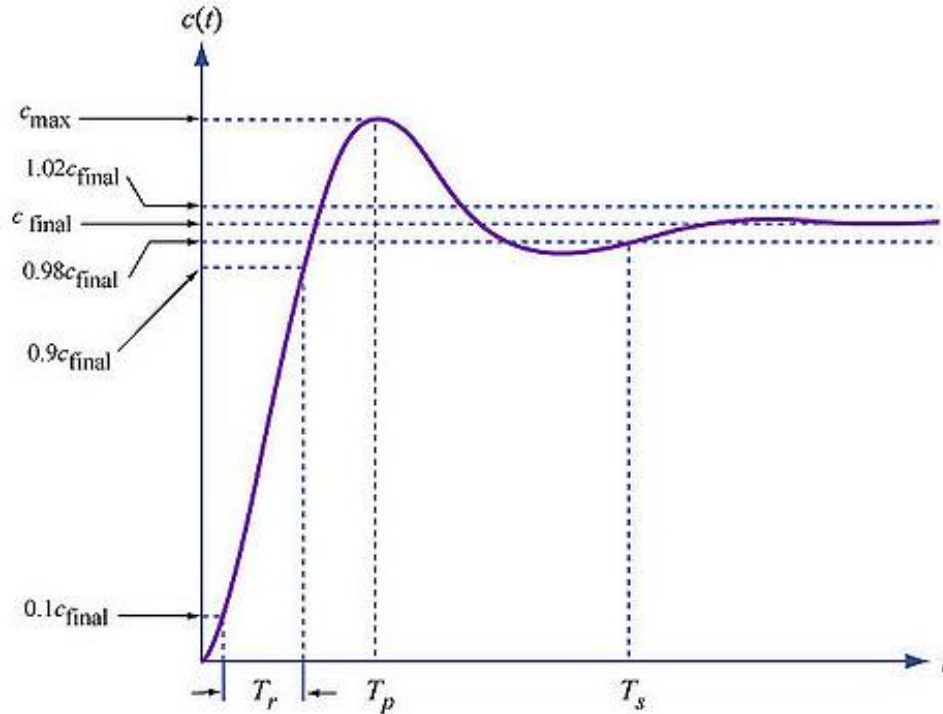
The amount that the waveform overshoots the steady state, or final value at the peak time, expressed as a percentage of the steady state value

$$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$$



$$\zeta = \frac{-\ln(\%OS / 100)}{\sqrt{\pi^2 + \ln^2(\%OS / 100)}}$$

RESPONSE OF AN UNDERDAMPED SECOND ORDER SYSTEM



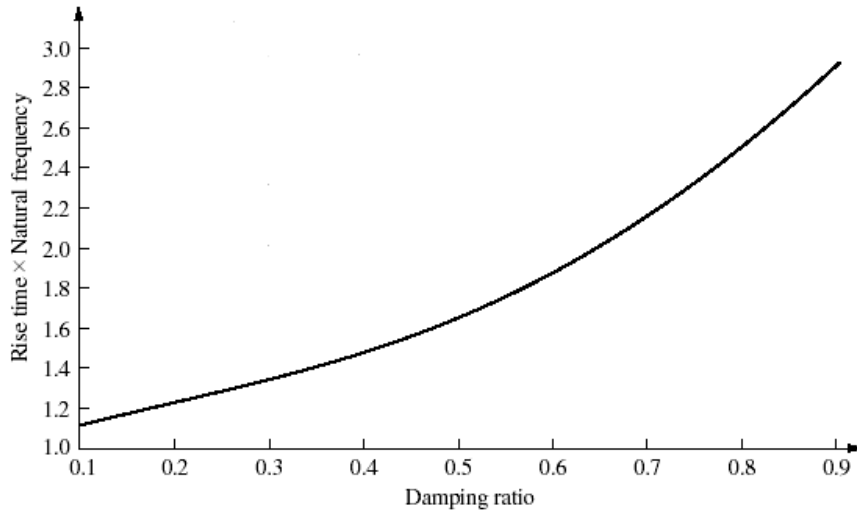
SETTLING TIME (T_s)

Time for the response to reach and stay within $\pm 2\%$ of the steady state value

$$T_s = \frac{4}{\zeta\omega_n}$$

Ref: N. S. Nise: Control Systems Engineering, 6th Ed., Wiley, 2013

RESPONSE OF AN UNDERDAMPED SECOND ORDER SYSTEM



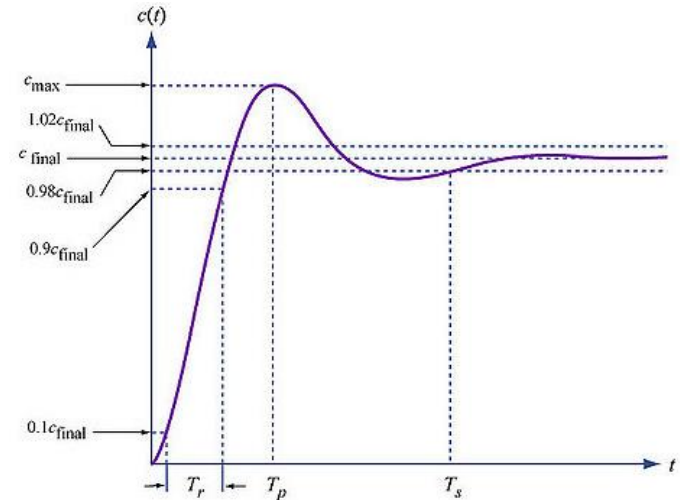
$$\omega_n T_r = 1.76 \zeta^3 - 0.417 \zeta^2 + 1.039 \zeta$$

$$\text{for } 0 < \zeta < 0.9$$

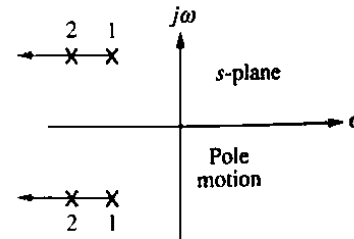
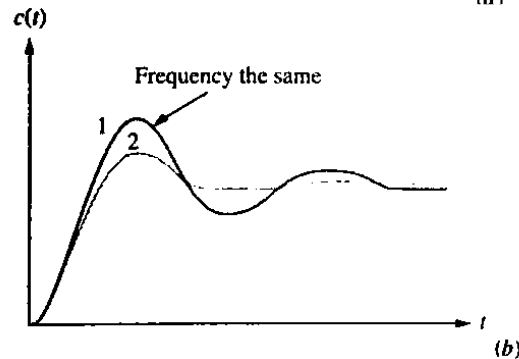
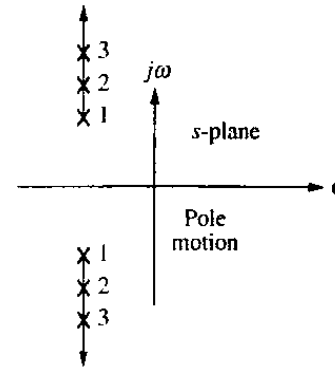
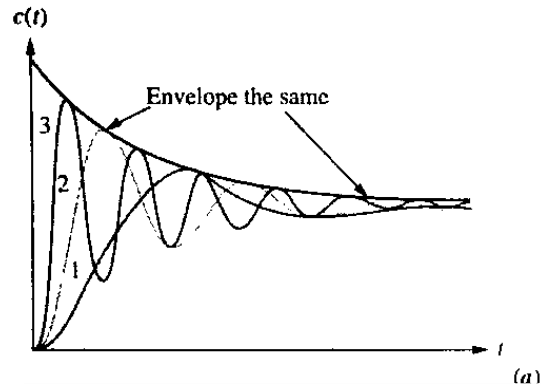
Ref: N. S. Nise: Control Systems Engineering, 6th Ed., Wiley, 2013

RISE TIME (T_r)

Time for the response to go from 0.1 to 0.9 of its final value

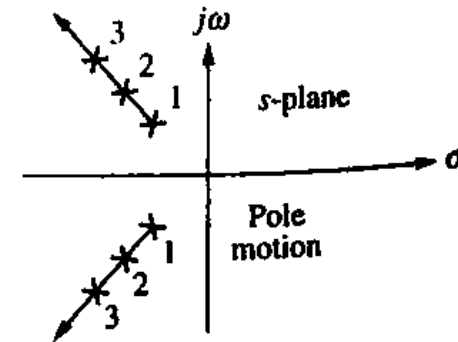
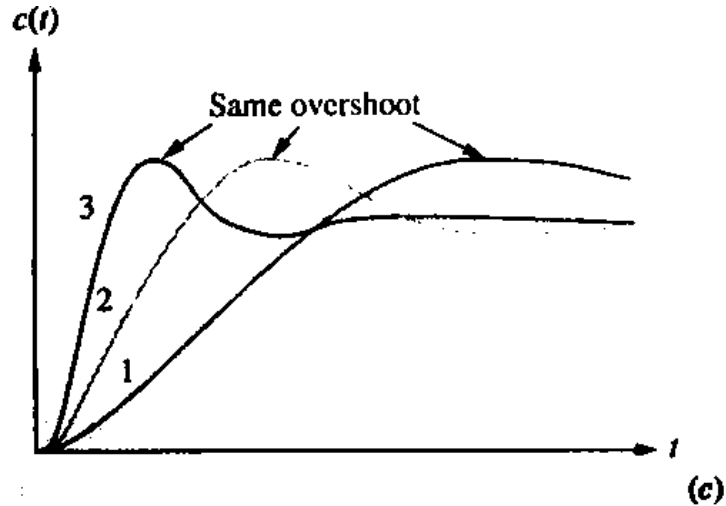


POLES MOVE WITH CONSTANT REAL AND IMAGINARY PART



Ref: N. S. Nise: Control Systems Engineering, 6th Ed., Wiley, 2013

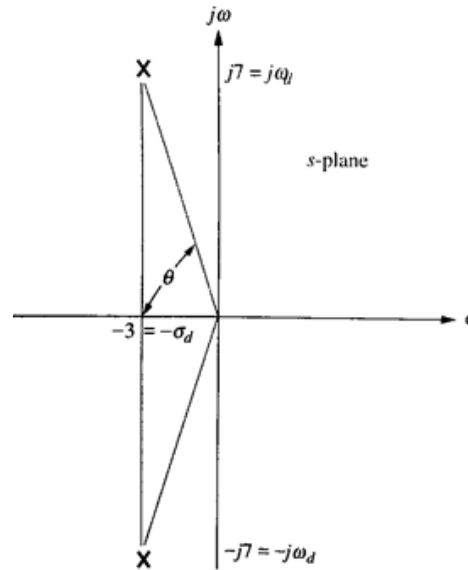
POLES MOVE IN RADIAL DIRECTION



Ref: N. S. Nise: Control Systems Engineering, 6th Ed., Wiley, 2013

NUMERICAL EXAMPLE

- Find damping, natural frequency, peak time, percent overshoot and settling time from the given pole plot.



Ref: N. S. Nise: Control Systems Engineering, 6th Ed., Wiley, 2013

REFERENCE BOOKS

- Norman S. Nise, Control Systems Engineering, Wiley, 2013
- Katsuhiko Ogata, Modern Control Engineering, Prentice Hall, 2010.

